RESEARCH NOTE

Horizontal Two-Phase Flow Across Tube Banks

I. D. R. Grant† and D. Chisholm†

Improved equations for predicting two-phase pressure drop in horizontal flow of two-phase mixtures across tube banks are presented.

NOTATION

B	Coefficient in equation (1)
D	Tube outside diameter
Dp_F	Pressure gradient for two-phase flow
$Dp_{F,LO}$	Pressure gradient if total mixture flows as liquid
Fr_{LO}	Froude number if all mixture flows as liquid
G	Mass velocity
N	Number of tube rows normal to flow
n	Blasius exponent
R	Term given by equation (16)
Re_{GO}	Reynolds number if total mixture flows as gas
Re_{LO}	Reynolds number if total mixture flows as liquid
U_G	Gas velocity
v_G	Specific volume of gas
v_L	Specific volume of liquid
X	Lockhart–Martinelli parameter
x	Mass dryness fraction
α	Ratio of gas to total cross-sectional area (void fraction)
Γ	Physical property coefficient, equation (5)
λ_{GO}	Friction factor corresponding to Re_{GO}
λ_{LO}	Friction factor corresponding to Re_{LO}
μ_G	Absolute viscosity of gas
μ_L	Absolute viscosity of liquid
ρ_G	Density of gas
ρ_L	Density of liquid
ϕ^2_{FL}	Two-phase multiplier, equation (7)
$\phi^2_{F,LO}$	Two-phase multiplier, equation (1)
ψ	Parameter defined by equation (12)
Ω	Term defined by equation (22)
ω	Term defined by equation (23)

1 INTRODUCTION

Two-phase flow across tube banks occurs in condensers, boilers, mist flow coolers, and similar types of equipment. A review of modern design methods for predicting the pressure gradients in these conditions is given in reference (1). In this note equations are developed which significantly improve the accuracy of prediction over existing methods.

† National Engineering Laboratory, East Kilbride, Glasgow.

Received 10 January 1980 and accepted for publication on 18 March 1980.

2 THE BASIC EQUATION

The equation used in a number of papers (2–4) for predicting two-phase pressure gradients is

$$\frac{Dp_F}{Dp_{F,LO}} = \phi^2_{F,LO} = 1 + (\Gamma^2 - 1) \times \{Bx^{(2-n)/2}(1-x)^{(2-n)/2} + x^{2-n}\} \tag{1}$$

where B is a coefficient which is either obtained from experiment or derived from theory. The Blasius exponent, n, is evaluated from

$$\frac{\lambda_{LO}}{\lambda_{GO}} = \left(\frac{Re_{GO}}{Re_{LO}}\right)^n \tag{2}$$

where the Reynolds numbers are defined as

$$Re_{GO} = GD/\mu_G \tag{3}$$

$$Re_{LO} = GD/\mu_L \tag{4}$$

and λ_{GO} and λ_{LO} are the corresponding friction factors. The physical property coefficient is

$$\Gamma^2 = \frac{\lambda_{GO}}{\lambda_{LO}}\frac{v_G}{v_L}. \tag{5}$$

3 THE B COEFFICIENTS

The zero interface shear model (5) can be approximated (6) by

$$B = \frac{2^{2-n} - 2}{\Gamma + 1} \tag{6}$$

and the 'pseudo-homogeneous' equation (7)

$$\phi^2_{FL} = 1 + \left\{\left(\frac{v_L}{v_G}\right)^{1/2} + \left(\frac{v_G}{v_L}\right)^{1/2}\right\}\frac{1}{X} + \frac{1}{X^2} \tag{7}$$

by (3)

$$B = \left(\frac{\mu_L}{\mu_G}\right)^{n/2}. \tag{8}$$

For the condition $n = 0$, homogeneous theory is given by

$$B = 1{\cdot}0 \tag{9}$$

in equation (1), which is consistent with equation (8).

In reference (1) the authors recommended for horizontal crossflow the following:

$$\text{Spray and bubbly flow } B = 0{\cdot}75 \tag{10}$$

0142/727X/80/0600-0097$02.00

Stratified and stratified spray $B = 0{\cdot}25$. (11)

For convenience we introduce the parameter (8) ψ such that

$$\frac{\phi^2_{F,LO} - 1}{\Gamma^2 - 1} = \psi \tag{12}$$

and note that equation (1) can then be written

$$\psi = Bx^{(2-n)/2}(1-x)^{(2-n)/2} + x^{2-n}. \tag{13}$$

4 AN IMPROVED PROCEDURE

Analysis of the available data (4) has shown that over a considerable range of conditions

$$\psi = Rx^{2-n} \tag{14}$$

where

$$R = 1{\cdot}3 + \frac{50}{10^6}G^2. \tag{15}$$

The units of G are kg/m^2 s. Making the equation dimensionless in terms of an all-liquid Froude number gives, taking the tube diameter (19·1 mm) as the characteristic length,

$$R = 1{\cdot}3 + 9{\cdot}45Fr_{LO}. \tag{16}$$

Where equations (14) and (15) give values of ψ in excess of equations (6) and (13), the former should be used; and where equations (14) and (15) give values of ψ in excess of equations (8) and (13), the latter should be used.

Figure 1 illustrates the comparison with experiment. Further experiments are required to justify the use of equation (8) (with (13)) as the upper envelope, though from Fig. 1a it can be seen to be applicable at high values of dryness fraction. Meanwhile in practice it should give a conservative (i.e., excessive) estimate of pressure drop.

The range of application of the equations is shown in Table 1 in terms of the following dryness fractions.

From equations (6), (13), and (14)

$$x_1 = \frac{1}{1 + \left\{\dfrac{(\Gamma+1)(R-1)}{2^{2-n}-2}\right\}^{2/(2-n)}} \tag{17}$$

and from equations (8), (13), and (14)

$$x_2 \doteqdot \frac{1}{1 + \left\{(R-1)\left(\dfrac{\mu_G}{\mu_L}\right)^{n/2}\right\}^{2/(2-n)}} \tag{18}$$

5 GENERALIZING THE FORMULA FOR R

An approximate formula is now obtained for the effects of the number of tube rows and physical properties.

Table 1

Range of Application of Equations

Dryness fraction x	Equations
$x < x_1$	(6) and (13)
$x_1 < x < x_2$	(14) and (16)
$x > x_2$	(8) and (13)

Assuming x_1 is a function of the vapour kinetic energy and the amount of separate liquid (there is no entrainment at x_1) then for a particular x_1

$$U_G\rho_G^{1/2} = U_{Gr}\rho_{Gr}^{1/2} \tag{19}$$

where the subscript r indicates characteristics in the reference tests quoted above ($N_r = 4$), and

$$(1-x)GN = (1-x_r)G_rN_r. \tag{20}$$

This assumes the flow area is proportional to the number of tube rows normal to flow.

The vapour-phase continuity equation is

$$xG = U_G\alpha\rho_G. \tag{21}$$

Define
$$\Omega = \alpha(\rho_G)^{1/2} \tag{22}$$

and
$$\omega = x + (1-x)\frac{N}{N_r}\frac{\Omega}{\Omega_r} \tag{23}$$

then from the above equations

$$x_r = \frac{x}{\omega} \tag{24}$$

and
$$G_r = G\omega\frac{\Omega_r}{\Omega}. \tag{25}$$

As the gas kinetic energy is the same the pressure gradient will be approximately constant

$$\mathrm{D}p_{F,LOr}\{1 + (\Gamma_r^2 - 1)\psi_r\} = \mathrm{D}p_{F,LOr}\left(\frac{G}{G_r}\right)^{2-n}\{1 + (\Gamma^2-1)\psi\}. \tag{26}$$

Hence
$$\psi \approx \psi_r\left(\frac{G_r}{G}\right)^{2-n}\frac{\Gamma_r^2}{\Gamma^2} \approx \psi_r\left(\frac{G_r}{G}\right)^{2-n}\frac{\rho_G}{\rho_{Gr}} \tag{27}$$

assuming that Γ^2 is large compared to unity, any variation in Γ^2 is due to density change, and the following term is negligible.

$$\left\{\left(\frac{G_r}{G}\right)^{2-n} - 1\right\}\frac{1}{\Gamma^2}$$

From equations (14), (16), (22), (24), (25), and (27)

$$R = \left\{1{\cdot}3 + 9{\cdot}45Fr_{LO}\left(\omega\frac{\Omega_r}{\Omega}\right)^2\right\}\left(\frac{\alpha_r}{\alpha}\right)^{2-n}\left(\frac{\rho_G}{\rho_{Gr}}\right)^{n/2}. \tag{28}$$

The analysis assumes stratified flow; for that reason it is only valid at higher void fractions. It is recommended therefore that the void fraction ratio in practice is taken as unity (this may overestimate the pressure drop). Also it is unlikely that the approximate nature of the analysis warrants inclusion of the small density ratio term giving finally

$$R = 1{\cdot}3 + 9{\cdot}45Fr_{LO}\left(\omega\frac{\Omega_r}{\Omega}\right)^2. \tag{29}$$

At smaller values of dryness fraction, noting that $N_r = 4$, this reduces to

$$R = 1{\cdot}3 + 0{\cdot}59Fr_{LO}N^2. \tag{30}$$

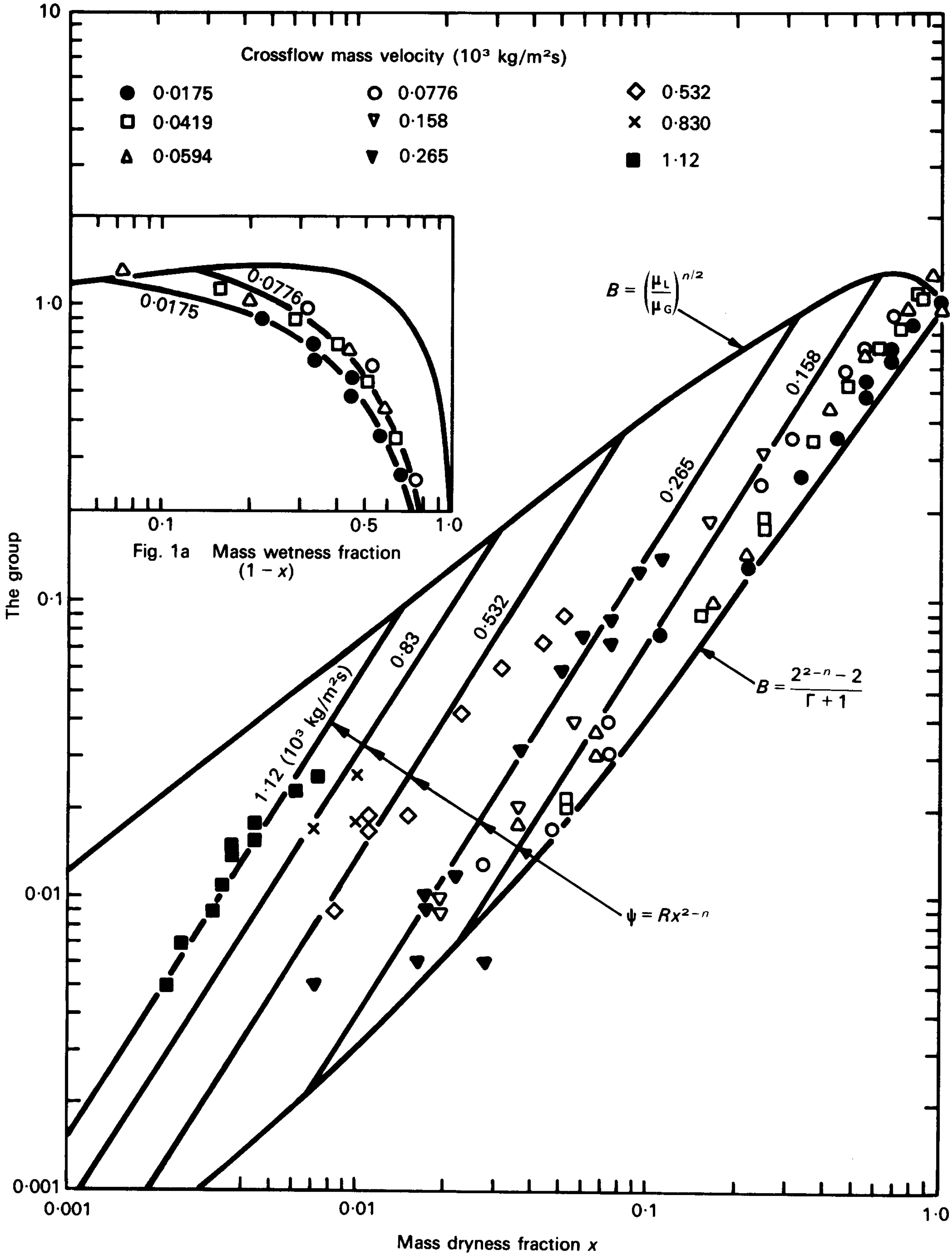

Fig. 1. Comparison of equations with experimental data

6 CONCLUSIONS

Table 1 summarizes a system of equations which give improved correlation of data for two-phase pressure drop in horizontal flow across tube banks.

ACKNOWLEDGEMENT

This paper is published by permission of the Director, National Engineering Laboratory, Department of Industry. It is Crown Copyright. The work reported was supported by The Chemical and Minerals Requirements Board of the Department of Industry.

REFERENCES

(1) GRANT, I. D. R., and CHISHOLM, D. 'Two-phase flow on the shell-side of a segmentally baffled shell-and-tube heat exchanger', *J. Heat Transfer*, 1979, **101**(1), 38–42

(2) CHISHOLM, D. 'Prediction of pressure drop at pipe fittings during two-phase flow', *Proc. 13th Int. Inst. Refrigeration Cong.*, Washington, DC, 27 Aug.–3 Sept. 1971, Vol. 2, 781–789

(3) CHISHOLM, D. 'Pressure gradients due to friction during the flow of evaporating two-phase mixtures in smooth tubes and channels', *Int. J. Heat Mass Transfer*, 1973, **16**, 347–358

(4) GRANT, I. D. R., and MURRAY, I. *Pressure drop on the shell-side of a segmentally baffled shell-and-tube heat exchanger with horizontal two-phase flow*. NEL Report No. 560, 1974. (East Kilbride, Glasgow: National Engineering Laboratory)

(5) GLOYER, W. 'Thermal design of mixed vapour condensers', *Hydrocarbon Process*, 1970, **49**(6), 103–108

(6) CHISHOLM, D. Discussion of paper 'A theoretical solution of the Lockhart and Martinelli flow model for calculating two-phase pressure drop and hold-up', by T. Johannessen, *Int. J. Heat Mass Transfer*, 1973, **16**, 225–226

(7) CHISHOLM, D. 'Pressure gradients during flow of incompressible two-phase mixtures through pipes, venturis and orifice plates', *Br. Chem. Engng.*, 1967, **12**(9), 1368–1371

(8) CHISHOLM, D. *Pressure gradients during the flow of evaporating two-phase mixtures*. NEL Report No. 470, 1970. (East Kilbride, Glasgow: National Engineering Laboratory)

(9) CHISHOLM, D., and SUTHERLAND, L. A. 'Prediction of pressure gradients in pipeline systems during two-phase flow', Symp. on Fluid Mechanics and Measurements in Two-phase Flow. *Proc. Instn. mech. Engrs.*, 1969–70, **184**(Pt 3C), 24–32